Title: Some fundamental aspects of the modelling of the interaction between atoms and photons with emphasis on ultrastrong coupling and the Dicke criticality

Abstract: We revisit how the Coulomb gauge and multipolar gauge of the quantum electrodynamics of atoms fare as starting points of standard models of quantum optics, like the Tavis-Cummings and Dicke models. We outline such a formulation of the multipolar gauge that is valid with an arbitrary configuration of boundary surfaces, that is, in the most general cavity qed situation. We point out that this gauge resolves the infamous A-square problem arising in the Coulomb gauge, that had been used as an argument against the feasibility of the Dicke phase transition. On the basis of the validity of the independent-atom picture, we give an upper limit for the collective coupling strength between atoms and the electromagnetic field in the ultrastrong-coupling regime, finding that the Dicke criticality coincides with solidification of the atomic medium.